Photon as a Symmetry-Breaking Solution to Field Theory. II*†

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A spin-zero field coupled to itself by a current-current interaction is examined subject to the requirements that the vacuum expectation of the current be nonvanishing. It is found that this theory is equivalent to the ordinary electrodynamics of a spinless particle. In all its broken-symmetry aspects the theory is similar to the Bjorken theory for Fermi particles, although the calculations necessary to arrive at this conclusion are more complex.

INTRODUCTION

IN the preceding paper[1] (hereafter referred to as I) we discussed the restrictions imposed on a theory where the vacuum expectation of a vector operator j^μ is required to be nonvanishing. As a specific example a Fermi field interacting with itself through a current-current interaction[2] was considered. It was found that this theory reproduced the electrodynamics of a spin-$\frac{1}{2}$ particle. This theory was particularly interesting and suggestive because it demonstrated that through the appropriate identification of terms an unrenormalizable theory involving one field can be transformed so as to be equivalent to a renormalizable theory involving two fields. In this paper we shall perform the analogous manipulation for a two-component self-coupled spinless field. These manipulations will lead to the normal electrodynamics of spinless particles. The underlying broken-symmetry structure will be found to be essentially identical to that found for the Bjorken model.

I. DERIVATION OF THE GREEN'S FUNCTIONS

The Lagrange density is taken to be

$$\mathcal{L}(x)=\phi^\mu\partial_\mu\phi+\tfrac{1}{2}\phi^\mu\phi_\mu-\tfrac{1}{2}m^2\phi\phi -\tfrac{1}{2}g_0[i\phi q\phi^\mu][i\phi q\phi_\mu]+J^\mu[i\phi q\phi_\mu]. \quad (1.1)$$

The two component fields ϕ and ϕ^μ are Hermitian and the matrix $q=\sigma_2$. From (1.1) we may derive the usual equal-time commutation relations

$$[\phi(x),\phi(x')]_{x'^0=x^0}=0 \quad [\phi(x),\phi^k(x')]_{x'^0=x^0}=0$$

and

$$(1/i)[\phi(x),\phi^0(x')]_{x_0=x_0'}=\delta^3(\mathbf{x}-\mathbf{x}')\,.$$

By varying ϕ^μ and ϕ in Eq. (1.1) we find the field equations

$$[\partial^\mu+g_0ij^\mu\cdot q-iJ^\mu q]\phi=-\phi^\mu \quad (1.2a)$$

and

$$[\partial^\mu+g_0ij^\mu\cdot q-iJ^\mu q]\phi_\mu+m^2\phi=0. \quad (1.2b)$$

In these equations the convenient definition $j^\mu\equiv i\phi q\phi^\mu$ has been made. It is easily verified that $\partial_\mu j^\mu=0$. We now break the Lorentz symmetry by imposing the condition

$$\left.\frac{\langle 0\sigma_1|j^\mu|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}\right|_{J=0}=\eta^\mu(x)|_{J=0}=\eta^\mu\neq 0. \quad (1.3)$$

The possibility of making this requirement hinges on the intrinsic ambiguity in the meaning of the product of two field operators at the same point in space-time. In electrodynamics this product is understood through a gauge-invariant averaging procedure which precludes the realization of (1.3). As was found in I, the concurrent validity of both current conservation and Eq. (1.3) is very dependent on how this equal time product is defined through a cutoff procedure in the Green's-function realization of (1.3). The same observations will be found to apply here.

If the new operator

$$D'^\mu\equiv\partial^\mu+g_0ij^\mu q-iJ^\mu q$$

is introduced, Eq. (1.2a) becomes

$$D'^\mu\phi=-\phi^\mu \quad (1.2c)$$

while Eq. (1.2b) becomes

$$D_\mu'\phi^\mu+m^2\phi=0. \quad (1.2d)$$

Combining these we find the familiar equation

$$[-D_\mu'D'^\mu+m^2]\phi=0. \quad (1.4)$$

Equation (1.4) establishes the equivalence of the usual first-order formalism[3] to the second-order formalism.

It is convenient to study the two boson propagators.

$$G(x,y)=i\frac{\langle 0\sigma_1|(\phi(x)\phi(y))_+|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle} \quad (1.5)$$

and

$$G^\mu(x,y)=i\frac{\langle 0\sigma_1|(\phi^\mu(x)\phi(y))_+|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}. \quad (1.6)$$

From the field equations it is found that these two

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[2] J. D. Bjorken, Ann. Phys. **24**, 174 (1963).

[3] Julian Schwinger, unpublished lecture notes, Harvard University.

propagators are related through the equation

$$G^\mu(x,y)=-\left[\partial_x{}^\mu-iJ^\mu(x)q+g_0i\eta^\mu(x)q+g_0iq\frac{-i\delta}{\delta J_\mu(x)}\right]G(x,y).\quad(1.7\text{a})$$

It we make the convenient definition

$$D_x{}'^\mu\left(\frac{-i\delta}{\delta J}\right)\equiv\left[\partial_x{}^\mu-iJ^\mu(x)q+g_0i\eta^\mu(x)q+g_0iq\frac{-i\delta}{\delta J_\mu(x)}\right]$$

Eq. (1.7a) may be written in the compact form

$$G^\mu(x,y)=-D_x{}'^\mu\left(\frac{-i\delta}{\delta J}\right)G(x,y).\quad(1.7\text{b})$$

The field equations and the commutation relations then require that

$$D_x{}'^\mu\left(\frac{-i\delta}{\delta J}\right)G_\mu(x,y)=\delta(x-y)-m^2G(x,y).\quad(1.8\text{a})$$

Combining (1.7b) and (1.8a) it is found, corresponding to Eq. (1.4), that

$$\left[-D'^\mu\left(\frac{-i\delta}{\delta J}\right)D_\mu'\left(\frac{-i\delta}{\delta J}\right)+m^2\right]G=1.\quad(1.9)$$

We now introduce the suggestive notation

$$A^\mu(x)\equiv g_0\frac{\langle 0\sigma_1|j^\mu(x)|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}-J^\mu(x)=g_0n^\mu(x)-J^\mu(x)$$

and define the function

$$D^{\mu\nu}(y,x)\equiv(\delta A^\nu(x)/\delta J^\mu(y))=-g^{\mu\nu}\delta(x-y)+g_0G^{\mu\nu}(x,y).\quad(1.10)$$

It will turn out that $D^{\mu\nu}(y,x)$ corresponds to the photon propagator of ordinary electrodynamics. Thus $D^{\mu\nu}(k)$ will be found to have a pole at $k^2=0$. In Eq. (1.10) the auxiliary function $G^{\mu\nu}(x,y)$ is defined as

$$G^{\mu\nu}(x,y)=i\frac{\langle 0\sigma_1|(j^\mu(x)j^\nu(y))_+|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}-\frac{i\langle 0\sigma_1|j^\mu(x)|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}\frac{\langle 0\sigma_1|j^\nu(y)|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}.$$

To facilitate this comparison to electrodynamics, derivatives with respect to the external source J^μ are replaced by derivatives with respect to the "vacuum vector potential" A^μ. The chain rule shows that

$$\frac{\delta}{\delta J^\mu(y)}=\frac{\delta A^\nu(x)}{\delta J^\mu(y)}\frac{\delta}{\delta A^\nu(x)}=D^{\mu\nu}(y,x)\frac{\delta}{\delta A^\nu(x)}.$$

With this device, Eq. (1.7a) may be written as

$$G^\mu(x,y)=-[\partial_x{}^\mu+iA^\mu q+g_0qD^{\mu\nu}(xy)(\delta/\delta A^\nu(y))]G(x,y)\quad(1.7\text{c})$$

while Eq. (1.8a) becomes

$$[\partial_x{}^\mu+iA^\mu q+g_0qD^{\mu\nu}(x,y)(\delta/\delta A^\nu(y))]G_\mu(x,y)=\delta(x-y)-m^2G(x,y).\quad(1.8\text{b})$$

In ordinary electrodynamics, corresponding to (1.7c) it is found that

$$G^\mu(x,y)=-[\partial_x{}^\mu+iA_e{}^\mu q+e_0qD_e{}^{\mu\nu}(xy)(\delta/\delta A_e{}^\nu(y))]G(x,y)$$

while corresponding to (1.8b), we find

$$[\partial_x{}^\mu+iA_e{}^\mu q+e_0qD_e{}^{\mu\nu}(x,y)(\delta/\delta A_e{}^\nu(y))]G_\mu(xy)=\delta(x-y)-m^2G(xy).$$

Here

$$A_e{}^\mu=\frac{\langle 0\sigma_1|A^\mu|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}$$

and

$$D_e{}^{\mu\nu}=i\frac{\langle 0\sigma_1|(A^\mu(x)A^\nu(y))_+|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}.$$

A^μ is the ordinary vector potential. It is thus clear that to within constant factors, Eqs. (1.7c) and (1.8b) are identical to the equations of electrodynamics if $D^{\mu\nu}$ has the form claimed. In the limit that $J^\mu=0$, (1.7c) and (1.8b) differ from ordinary electrodynamics in as much as $(A^\mu)_{J=0}=g_0\eta^\mu$, while $(A_e{}^\mu)_{J=0}=0$. This is of no concern, as a constant potential has no physical effect.

For the construction of $D^{\mu\nu}$ we consider the lowest approximations to G and G^μ. These are found by neglecting the variational derivatives in (1.7a) and (1.8a). The basic equations are then

$$G^\mu(x,y)=-[\partial_x{}^\mu-iJ^\mu(x)q+ig_0q\eta^\mu(x)]G(xy)=-D^\mu(x,\xi)G(\xi,y)\quad(1.11\text{a})$$

and

$$[\partial_x{}^\mu-iJ^\mu(x)q+ig_0q\eta^\mu(x)]G_\mu(x,y)=D^\mu(x\xi)G_\mu(\xi y)=\delta(x-y)-m^2G(x,y).\quad(1.12)$$

Here, the quantity D^μ is

$$D^\mu(x,\xi)=[\partial_x{}^\mu(x-\xi)-iJ^\mu(\xi)q\delta(x-\xi)+ig_0q\eta^\mu(\xi)\delta(x-\xi)].$$

Of course, the spatial indices x and ξ of $D^\mu(x,\xi)$ will be left implicit in most of what follows.

If (1.11a) is inserted into (1.12), we find that

$$G=1/(-D^\alpha D_\alpha+m^2).\quad(1.13\text{a})$$

Inserting (1.13a) back into (1.11a) yields

$$G^\mu=-D^\mu[1/(D^\alpha D_\alpha+m^2)].\quad(1.13\text{b})$$

If we represent G in the form

$$G(x,y)|_{J=0}=\int e^{ip(x-y)}G(p)d^4p\,, \tag{1.14}$$

it follows that

$$G^\mu(x,y)|_{J=0}=\int -(ip^\mu+ig_0q\eta^\mu)G(p)e^{ip(x-y)}\,. \tag{1.15}$$

Equation (1.13a) then shows that

$$G(p)=(2\pi)^{-4}[(p^\alpha+g_0q\eta^\alpha)(p_\alpha+g_0q\eta_\alpha)+m^2]^{-1}\,. \tag{1.16a}$$

A representation equivalent to (1.16a) that has all of the q dependence in the numerator is

$$G(p)=\frac{1}{(2\pi)^4 i}\frac{(p^\mu+g_0q\eta^\mu)([p^2+m^2+g_0^2\eta^2]-2g_0q(\eta\cdot p))}{[p^2+m^2+g_0^2\eta^2]^2-4g_0^2(\eta\cdot p)^2}\,. \tag{1.17}$$

Since $j^\mu=i\phi q\phi^\mu$, the condition of broken symmetry (1.3) may be written as

$$\eta^\mu=\mathrm{tr}qG^\mu(xx) \tag{1.18}$$

which, with the aid of Eq. (1.17), becomes

$$\eta^\mu=\frac{2g_0\eta_\nu}{i}\int\frac{d^4p}{(2\pi)^4}\frac{[g^{\mu\nu}[p^2+m^2+g_0^2\eta^2]-2p^\mu p^\nu]}{[p^2+m^2+g_0^2\eta^2]^2-4g_0^2(\eta\cdot p)^2}\,. \tag{1.19}$$

If the factor $2g_0$ on the right-hand side of Eq. (1.19) were replaced by $8g_0$, this equation would be identical to the condition equation for the fermion model of I. This factor of 4 difference between the two models is trivial and only reflects the fact that the fermion has additional spin degrees of freedom. Because of this trivial difference all the discussion and equations of I relevant to the convergence and structure of the condition integral are true in the boson case provided that, in the equations, we take account of the factor of 4.

In particular, if we introduce a cutoff and make the definitions

$$Q=\frac{8g_0}{(2\pi)^4 i}\int\frac{d^4p}{p^2+m^2}\,,\quad L=\frac{8g_0}{i(2\pi)^4}\int\frac{d^4p}{[p^2+m^2]^2}$$

and

$$F=\frac{8g_0}{(2\pi)^4 i}\int\frac{d^4p}{[p^2+m^2]^3}$$

it follows that

$$\mathrm{Tr}qG^\mu(xx)=(\tfrac{1}{8}Q+\tfrac{1}{8}m^2L-\tfrac{1}{12}g_0^2\eta^2F)\eta^\mu \tag{1.20a}$$

and hence that the condition (1.18) for $\eta\neq0$ is

$$1=\tfrac{1}{8}Q+\tfrac{1}{8}m^2L-\tfrac{1}{12}g_0^2\eta^2F\,. \tag{1.20b}$$

In this model when the cutoff $\Lambda\to\infty$ Eq. (1.20b) becomes

$$1=g_0\Lambda^2/16\pi^2\,.$$

The ground work is now prepared for the study of the photon propagator. It is easily found that

$$(-i\delta/\delta J_\nu(y))D^\mu(x,\xi)=\delta(x-\xi)qD^{\nu\mu}(y,\xi)\,. \tag{1.21}$$

Differentiation of Eq. (1.13a) for G and the application of (1.21) results in the equation

$$(-i\delta/\delta J_\nu)G|_{J=0}=-D^{\nu\alpha}[GqG_\alpha+G_\alpha qG]\,. \tag{1.22}$$

In the same manner it is found that

$$-i\delta G_\mu/\delta J_\nu|_{J^\mu=0}=-D^{\nu\alpha}\{g_{\alpha\mu}qG-D_\mu[G_qG_\alpha+G_\alpha qG]\}\,. \tag{1.23}$$

Since

$$G^{\nu\mu}=i\,\mathrm{Tr}q(-i\delta G^\mu/\delta J_\nu)$$

we find that when $J^\mu=0$,

$$G_\mu{}^\nu=-iD^{\nu\alpha}[g_{\alpha\mu}\,\mathrm{Tr}G-\mathrm{Tr}D_\mu(GG_\alpha+G_\alpha G)]\,. \tag{1.24}$$

Inserting this into

$$(D^{\mu\nu}+g^{\mu\nu})/g_0=G^{\mu\nu}$$

with $J^\mu=0$ yields

$$D^{\nu\alpha}(z-x)\Big(g_{\alpha\mu}\delta(x-y)-\frac{g_0}{i}\{g_{\alpha\mu}\delta(x-y)\,\mathrm{Tr}G(x,x)+\mathrm{Tr}G_\mu(y-x)G_\alpha(x-y)-\mathrm{Tr}[D_\mu(y-\xi)G_\alpha(\xi-x)G(x-y)]\}\Big)=-g^\nu{}_\mu\delta(z-y)\,. \tag{1.25}$$

Introducing the Fourier representation

$$D^{\nu\alpha}(z-x)=\int e^{ik(z-x)}D^{\nu\alpha}(k)\,,$$

this becomes

$$[D^{\alpha\mu}(k)]^{-1}=(2\pi)^4\Big\{-g^{\alpha\mu}+\frac{g_0(2\pi)^4}{i}\Big[\int d^4p\Big(\mathrm{Tr}\frac{G(p)g^{\alpha\mu}}{(2\pi)^4}+\mathrm{Tr}[G^\mu(p)G^\alpha(p+k)-D^\mu(p)G^\alpha(p)G(p+k)]\Big)\Big]\Big\}\,. \tag{1.25b}$$

To invert, it is necessary to perform specifically the integrations in this relation. This is done essentially in the same manner as in I. The convenient shorthand $\pi'^{\alpha\mu}(k)$ is introduced through the definition

$$\pi'^{\alpha\mu}(k)\equiv\frac{g_0(2\pi)^4}{i}\int d^4p\Big(\frac{g^{\alpha\mu}}{(2\pi)^4}\mathrm{Tr}G(p)+\mathrm{Tr}[G^\mu(p)G^\alpha(p+k)-D^\mu(p)G^\alpha(p)G(p+k)]\Big)\,. \tag{1.26}$$

The zero energy part of $\pi'^{\alpha\mu}$ is related to the basic calculation (1.20) of the theory through the Ward's identity

$$\pi'^{\alpha\mu}(0)=\frac{-i\partial}{\partial\eta_\alpha}\Big[\int\frac{d^4p}{(2\pi)^4}\times\mathrm{Tr}\Big(q\frac{(p^\mu+g_0q\eta^\mu)}{(p^\lambda+g_0q\eta^\lambda)(p_\lambda+g^0q\eta_\lambda)+m^2}\Big)\Big]=\frac{\partial}{\partial\eta_\alpha}\mathrm{Tr}qG^\mu(xx).$$

Combining this equation with (1.20a) and the condition (1.20b) it follows that

$$\pi'^{\alpha\mu}(0)=\frac{\partial}{\partial\eta_\alpha}\mathrm{Tr}qG^\mu(xx)=g^{\alpha\mu}-(\tfrac{1}{6}g_0{}^2F)\eta^\alpha\eta^\mu=g^{\alpha\mu}-\tfrac{1}{4}C\eta^\alpha\eta^\mu.\quad(1.27)$$

Here, we have made the definition $C\equiv\frac{2}{3}Fg_0{}^2$. It may easily be seen that $\pi'^{\alpha\mu}(k)-\pi'^{\alpha\mu}(0)$ is independent of η^μ. A direct calculation shows for Λ/m large that

$$\pi'^{\alpha\mu}(k)=(g^{\mu\alpha}k^2-k^\mu k^\alpha)\bar{I}'(k^2)+g^{\alpha\mu}-\tfrac{1}{4}C\eta^\alpha\eta^\mu.\quad(1.28)$$

Here $\bar{I}'(k^2)$ is the second-order "photon mass" of ordinary scalar electrodynamics and is given by

$$\bar{I}'(k^2)=\frac{1}{24}\Big[L+\frac{g_0k^2}{2}\int_{4m^2}^\infty\frac{dK^2[1-4m^2/K^2]^{3/2}}{K^2[K^2+k^2-i\epsilon]}\Big].$$

It follows that

$$[D^{\alpha\mu}(k)]^{-1}=(2\pi)^4[(g^{\mu\alpha}k^2-k^\alpha k^\mu)\bar{I}'(k^2)-\tfrac{1}{4}C\eta^\alpha\eta^\mu].\quad(1.29)$$

It is now necessary to make the same decision as in I. The last term of this equation, which is of entirely different structure from the others, comes from taking the cutoff procedure very seriously in the calculation of (1.20a). If we did not regard this procedure so seriously, the last term would not have occurred. Inverting (1.29) with the term results in

$$D^{\alpha\mu}(k)=\frac{1}{(2\pi)^4}\Big[\frac{1}{k^2\bar{I}'(k^2)}\Big(\bar{g}^{\alpha\mu}+\frac{\eta^2k^\alpha k^\mu}{(\eta\cdot k)^2}\Big)-\frac{4k^\alpha k^\mu}{C(\eta\cdot k)^2}\Big].\quad(1.30)$$

Here the convenient notation

$$\bar{g}^{\alpha\mu}=g^{\alpha\mu}-k^\alpha\eta^\mu/\eta\cdot k-\eta^\alpha k^\mu/\eta\cdot k$$

has been introduced. Thus $D^{\alpha\mu}(k)$ describes the propagation of a zero mass photon. This propagator is identical in gauge structure to the propagator of I. Since it arises from currents in essentially the same way, this form is subject to the same difficulty with current conservation. If the cutoff is treated in a less literal manner we only retain the divergent terms of (1.20) and the last term of Eq. (1.28) does not occur. It is then possible to conclude that

$$D^{\alpha\mu}(k)=\frac{1}{(2\pi)^4}\frac{1}{k^2\bar{I}'(k^2)}\Big[g^{\alpha\mu}-\frac{k^\alpha k^\mu}{k^2}\Big].\quad(1.31)$$

This propagator is transverse for all values of k and consequently is consistent with current conservation.

It is easily found that for either of these propagators electrodynamics in a constant external field is reproduced, if the identification $\alpha=24g_0/L=24\pi^2/\mathrm{Im}(\Lambda/m)$ is made. This restriction is consistent with large cutoff Λ and small coupling constant g_0.

In conclusion, then, we have found that within the framework of the self-coupled charged-boson model it is possible to extract a photon without ever inserting a photon field operator A^μ. The propagator for this photon differs at most from the ordinary second-order electrodynamic propagator by gauge terms. We are now prepared to undertake the task of checking the consistency of this theory with the operator symmetries required by Lorentz invariance.

II. CONSISTENCY WITH ROTATIONS

As an expression of the vector nature of the operator j^μ, it is necessary that

$$(1/i)[J^{\mu\nu},j^\lambda(x)]=(x^\mu\partial^\nu-x^\nu\partial^\mu)j^\lambda(x)+g^{\mu\lambda}j^\nu-g^{\lambda\nu}j^\mu(x).$$

The application of condition (1.3) to the vacuum expectation of this equation yields

$$(1/i)\langle0|[J^{\mu\nu},j^\lambda(x)]|0\rangle=g^{\mu\lambda}\eta^\nu-g^{\lambda\nu}\eta^\mu.$$

With the aid of the relation

$$J^{\mu\nu}=\int d^3y[y^\mu T^{0\nu}(y)-y^\nu T^{0\mu}(y)]$$

and the introduction of the quantity

$$C_\eta{}^{\mu\nu\lambda}(y-x)=i\langle0|[T^{\mu\nu}(y),j^\lambda(x)]|0\rangle,$$

this equation is equivalent to the two equations

$$\int d^3yy^kC_\eta{}^{00\lambda}(y)=g^{0\lambda}\eta^k-g^{\lambda k}\eta^0\quad(2.1\mathrm{a})$$

and

$$\int d^3y[y^kC_\eta{}^{01\lambda}(y)-y^1C_\eta{}^{0k\lambda}(y)]=g^{1\lambda}\eta^k-g^{\lambda k}\eta^1.\quad(2.1\mathrm{b})$$

The analysis of these structures with the two forms of the photon propagator (1.30) or (1.31) was completely developed in I. It was found with the Bjorken form of the propagator (1.30) when $\eta^2=0$ that

$$C_\eta{}^{\mu\nu\lambda}(k)=[(C_1/i)\epsilon(k^0)\delta(k^2)\bar{g}^{\lambda(\mu}k^{\nu)}\eta\cdot k+(C_2/i)\delta(k\cdot\eta)k^\lambda\eta^\mu\eta^\nu].\quad(2.2)$$

It is necessary that $C_1 - C_2 = 1/(2\pi)^3$. The term proportional to $\delta(\eta\cdot k)$ is the only one that is present if $\eta^2 \neq 0$, and originates from the term proportional to $k^\lambda k^\beta/(\eta\cdot k)^2$ in the photon propagator. Its presence emphasizes the current nonconserving aspects of the Bjorken approximation. The tensor structure $k^\lambda \eta^\mu \eta^\nu$ has been set so as to assure energy-momentum conservation. It is, in fact, unlikely that an approximation which violates current conservation would respect energy-momentum conservation. However, since the $\eta^\mu\eta^\nu$ terms do not all originate in inverse propagator structures, and will be shown to be immune from exact calculation, we exercise the prerogative of respecting this symmetry. We do not contend that the presence of this term should be taken seriously in interpreting broken symmetry phenomena since its origin is due primarily to a symmetry-destroying approximation. Nevertheless, its presence is reasonable in a charge-nonconserving theory and very suggestive. The term $\delta(\eta\cdot k)$ in no sense corresponds to a normal single-particle excitation in a Lorentz invariant theory. It must be interpreted as representing transitions mediated by j^μ between the standard vacuum and states built on other vacuums whose occurence is guaranteed by the broken symmetry requirement (1.3). This then is one type of spurious transition related to those suggested by Klein and Lee.[4] However, it is clear that its origin is inextricably bound to the presence of the zero mass particle of the theory. As proposed in I, we feel that this might be indicative that the statement made by the Goldstone theorem[5] is always correct in fully relativistic theories, although its proof restricted to normal spectral weights is not sufficiently general. For the Lorentz gauge form of $D^{\alpha\mu}(k)$ given by (1.31) it is necessary that

$$C_\eta^{\mu\nu\lambda}(k) = (C_4/i)\epsilon(k^0)\delta(k^2)k^\lambda \bar{g}^{\mu\nu}(n\cdot k). \tag{2.3}$$

Direct calculation with Eq. (2.1) shows that

$$C_4 = 1/(2\pi)^3. \tag{2.4}$$

It is the responsibility of a consistent calculational procedure to explicitly verify Eq. (2.4).

To perform this calculation we introduce the symmetrical energy momentum tensor corresponding to (1.1). It is easily shown that this is

$$T^{\mu\nu} = \phi^\mu\phi^\nu - g_0 j^\mu j^\nu + 2J^{(\mu}j^{\nu)} - \tfrac{1}{2}g^{\mu\nu}[\phi^\alpha\phi_\alpha - g_0 j^\alpha j_\alpha + 2J^\alpha j_\alpha + m^2\phi^2]. \tag{2.5}$$

It may be checked by use of the field equations that

$$\partial_\mu T^{\mu\nu}(x) = 0.$$

Using the definitions made in Sec. I for G, D^μ, and G^μ it is found that

$$\begin{aligned}\frac{i\langle 0\sigma_1|T^{\mu\nu}(y)|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle} =& \operatorname{Tr}[G^{(\mu}(y,\xi)D^{\nu)}(\xi,p)]_{p\to y-0} - g_0 i \operatorname{Tr}qG^\mu(y,y)\operatorname{Tr}qG^\nu(y,y) + 2iJ^{(\mu}\operatorname{Tr}qG^{\nu)}(y,y) \\ &+ \tfrac{1}{2}g^{\mu\nu}[\operatorname{Tr}[G^\alpha(y,\xi)D_\alpha(\xi p)]_{p\to y-0} - g_0 i[\operatorname{Tr}qG^\alpha(yy)][\operatorname{Tr}qG_\alpha(yy)] + 2iJ^\alpha \operatorname{Tr}qG_\alpha(yy) + m^2\operatorname{Tr}G(yy)] \\ &- g_0 i(-i\delta/\delta J^{(\mu})\operatorname{Tr}qG^{\nu)} + \tfrac{1}{2}g_0 i g^{\mu\nu}(-i\delta/\delta J^\alpha)\operatorname{Tr}qG^\alpha. \end{aligned} \tag{2.6}$$

Hereafter, the last two terms of Eq. (2.6) will be dropped. This is done in order to make this calculation consistent with the approximations used to determine G and G^μ in Sec. I.

It is convenient to determine $C_\eta^{\mu\nu\lambda}$ through the function

$$\begin{aligned} T_\eta^{\mu\nu\lambda}(y-x) &= i\left[\frac{-i\delta}{\delta J^\lambda(x)}\frac{\langle 0\sigma_1|T^{\mu\nu}(y)|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle} + \frac{i\langle 0\sigma_1|\delta T^{\mu\nu}(y)/\delta J^\lambda(x)|0\sigma_2\rangle}{\langle 0\sigma_1|0\sigma_2\rangle}\right]_{J=0} \\ &= i\langle 0|(T^{\mu\nu}(y)j^\lambda(x))_+|0\rangle - i\langle 0|T^{\mu\nu}(y)|0\rangle\eta^\lambda \end{aligned}$$

by use of the simple procedure outlined in I for transforming time-ordered products into commutators. From Eq. (2.6) we find, with the assistance of the equivalents to Eqs. (1.21), (1.22), and (1.23), with $J^\mu \neq 0$ that

$$\begin{aligned} T_\eta^{\mu\nu\lambda}(y-x) =& D^{\lambda B}(x-z)[2\delta(z-y)g_B{}^{(\mu}\eta^{\nu)} + \operatorname{Tr}D^{(\mu}(y,\alpha)[G(\alpha,z)qG_B(z,\xi) + G_B(\alpha,z)qG(z,\xi)]D^{\nu)}(\xi,y) \\ &+ 2g_0 i\eta^{(\nu}\left\{\operatorname{Tr}\{g^{\mu)}{}_B\delta(z-y)G(y,y) - D^{\mu)}(y,\alpha)[G(\alpha,z)G_B(z,y) + G_B(\alpha,z)G(z,y)]\} - \tfrac{1}{2}\frac{g^{\mu\nu}}{(2\pi)^4}Z_B(z-y)\right\}. \end{aligned} \tag{2.7}$$

In Eq. (2.7) the function $Z^B(z,y)$ is defined as

$$\begin{aligned} Z_B(z-y)/(2\pi)^4 =& [2\delta(z-y)\eta_B + \operatorname{Tr}D^\alpha(y,\alpha)[G(\delta,z)qG_B(z,\xi) + G_B(\delta,z)qG(z,\xi)]D_\alpha(\xi,y) + 2g_0 i\eta^\alpha \\ &\times \operatorname{Tr}\{g_{\alpha B}\delta(z-y)G(y,y) - D_\alpha(y,\delta)[G(\delta,z)G_B(z,y) + G_B(\delta,z)G(z,y)]\} - m^2\operatorname{Tr}[G(y,z)qG_B(z,y) + G_B(y,z)qG(z,y)]. \end{aligned}$$

Except for the term proportional to m^2, Z^B is just the trace on (μ,ν) of the terms preceeding it in Eq. (2.7). If the

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Fourier transform of Eq. (2.7) is taken, it follows that

$$T_\eta{}^{\mu\nu\lambda}(k)=D^{\lambda B}(k)\bigg[2g_B{}^{(\mu}\eta^{\nu)}+(2\pi)^4\int \mathrm{Tr}(D^{(\mu}(p)[G(p)qG_B(p+k)+G_B(p)qG(p+k)]D^{\nu}(p+k)d^4p$$
$$-2\eta^{(\nu}\pi_B{}'^{\mu)}(k)-\tfrac{1}{2}g^{\mu\nu}Z_B(k)]\equiv D(k)^{\lambda B}T_B{}^{\mu\nu}(k). \quad (2.8)$$

We know the structure of all the terms appearing in Eq. (2.8) except for the one of the form

$$(2\pi)^4\int d^4p[D^\mu(p)[G(p)qG_B(p+k)+G_B(p)qG(p+k)]D^\nu(p+k)\equiv E'^{B\mu\nu}(k)$$

and the ones proportional to m^2 in $Z_B(k)$. The terms proportional to m^2 have no momentum dependence and hence are only of significance to $T_B{}^{\mu\nu}(0)$.

By insertion of the forms for G^μ and G found in Sec. I, it is found that $E'^{B\mu\nu}(k)$ is quadratically divergent as $\Lambda\rightarrow\infty$, and odd in η^μ. A tedious calculation in which the cutoff is taken seriously by retaining all finite terms which occur with divergent terms yields

$$E'^{B\mu\nu}(k)=E'^{B\mu\nu}(0)+\tfrac{1}{6}F[\eta^B g^{\mu\nu}k^2+2\eta^B k^\mu k^\nu$$
$$+2k^2 g^{B(\mu}\eta^{\nu)}-2k^B k^{(\mu}\eta^{\nu)}]. \quad (2.9)$$

This expression has no part which looks like $\eta\cdot kg^{B(\mu}k^{\nu)}$ and consequently can make no contribution toward the satisfaction of Eq. (2.2). It takes very little further consideration to demonstrate that all contributions to these equations must come from the anomalous parts of $D^{\lambda B}(k)$ in the form

$$-4k^\lambda k^B/C(2\pi)^4(\eta\cdot k)^2 T_B{}^{\mu\nu}(0).$$

As in I, $T_B{}^{\mu\nu}(0)$ involves quadratically divergent terms which cannot be calculated from (1.20a) through any Ward's identity. Thus $T_B{}^{\mu\nu}(0)$ must be adjusted in order to guarantee the satisfaction of Eqs. (2.2). We therefore conclude that for η^μ time- or light-like that the relevant terms of $C_\eta{}^{\mu\nu\lambda}(k)$ are

$$C_\eta{}^{\mu\nu\lambda}(k)=[i/(2\pi)^3]\delta(\eta\cdot k)k^\lambda\eta^\mu\eta^\nu.$$

Thus the consistency, using propagator (1.30), occurs in a rather strained manner.

It is, however, very simple to check the consistency when finite terms occuring with divergent integrals are neglected and $D^{\lambda B}(k)$ is in the Lorentz gauge. Then (2.9) shows that

$$E'^{B\mu\nu}(k)=E'^{B\mu\nu}(0).$$

By the arguments of I, the momentum-independent parts of $T_B{}^{\mu\nu}(k)$ must vanish in this case, so we conclude from Eq. (2.8) that

$$T_\eta{}^{\mu\nu\lambda}(k)=D^{\lambda B}(k)[-2\eta^{(\nu}[\pi_B{}'^{\mu)}(k)-\pi_B{}'^{\mu)}(0)]$$
$$+g^{\mu\nu}\eta_\alpha[\pi'^\alpha{}_B(k)-\pi_B{}'^\alpha(0)].$$

Using (1.28) and the recipes of I for conversion of a time-ordered product into a commutator, (2.10) yields

$$C_\eta{}^{\mu\nu\lambda}(k)=[1/(2\pi)^3 i]k^\lambda\bar{g}^{\mu\nu}\eta\cdot k\epsilon(k^0)\delta(k^2).$$

This is in accordance with (2.3) and (2.4) and hence we may conclude that in the Lorentz gauge the scalar theory is consistent in an unstrained manner. Equation (2.10) clearly illustrates a point which was not so clear for the model of I. Namely, the consistency of the theory is guaranteed by the zero mass part of the current-current commutation relations of a conserved current.

In conclusion, we should like to point out that the essence of what we have done here can be reproduced in ordinary electrodynamics in the presence of a constant external potential. This is equivalent to imposing the broken-symmetry-like gauge requirement $\langle 0|A^\mu|0\rangle=\eta^\mu$. However, because of the gauge structure of Lorentz gauge electrodynamics, no general proof of the vanishing of the photon mass results from this procedure.[6]

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[6] G. S. Guralnik and C. R. Hagen (to be published).